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Mill Hill’s Calculation Overview

**Principles of the Policy**

One of the key learning principles behind this policy is the **concrete** (objects), **pictorial**, **abstract** (numbers) approach (CPA). The concrete-pictorial-abstract approach, is based on research by psychologist **Jerome Bruner**, and suggests that there are three steps (or representations) necessary for children to develop understanding of a concept.

For children to have a deep understanding of the mathematical concepts being developed, they need to ‘master’ all three phases of the CPA approach. If a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial as an additional scaffold. If a child is working at the abstract stage, ‘proving’ something or ‘working out’ then concrete or pictorial representations could be used to develop a greater depth as pupils articulate their thinking /reasoning. Reinforcement is achieved by going back and forth between these representations. Linking abstract notation to pictorial/concrete representations and then the concrete/pictorial models to an abstract notation. The White Rose Hub maths scheme is used to deliver this approach.

**Concrete Representation**

The ***enactive stage***. Children are first introduced to an idea/skill/concept by acting it out with real objects, this could include large scale with the pupils themselves and also utilise resourced available in the outdoor environment. This is a ***'hands on'*** stage using real objects linked to real-life and the wider curriculum and/or mathematical equipment, (i.e. counters, cubes, bead string, five and ten frames, Numicon, Dienes, place value counters etc.) and it is the foundation for ***CONCEPTUAL UNDERSTANDING***.

**Pictorial Representation**

The ***iconic stage***. A child has sufficiently understood the hands-on, ***CONCRETE*** experiences performed and can now relate them to ***PICTORIAL*** representations, such as a ***DIAGRAM*** or ***PICTURES*** of the problem. ***PICTORIAL*** representations, such as the bar model, can also be used to ***scaffold*** understanding.

**Abstract Representation**

The ***symbolic stage***. A child is now capable of representing problems by using ***ABSTRACT*** mathematical notation, for example: 12 ÷ 2 = 6. This is the ultimate mode.

Teachers should understand how each stage of the CPA approach can be used effectively to ***MODEL*** concepts, ***SCAFFOLD*** learning and ***RECORD*** thinking:

***MODELLING***: teachers to make clear links are made between ***CONCRETE*** representations (which can also be represented pictorially), ***PICTORIAL*** representations (diagrams and pictures, including bar modelling) and ***ABSTRACT*** notations.

***SCAFFOLDS***: children to be offered the opportunity to use appropriate ***CONCRETE*** and ***PICTORIAL*** representations to further ***scaffold*** their understanding. The ***scaffolds*** offered, must be familiar and understood by children. Children should be encouraged to consider whether ***scaffolds*** are required and for how long they require them for. ***CONCRETE*** and ***PICTORIAL*** representations are also supportive when developing children’s depth of knowledge through problem solving and reasoning experiences.

***RECORDING*:**

***CONCRETE RECORDING***: when children are unable to ***record*** their thinking using ***PICTORIAL*** representation or ***ABSTRACT*** notation then their learning can be evidenced through photographic evidence and post-it notes, which detail key information regarding children’s strengths and areas of developments/gaps/misconceptions.

***PICTORIAL RECORDING***: children to be encouraged to represent their thinking using ***PICTORIAL*** representations, if they are unable to record using ***ABSTRACT*** notation. ***PICTORIAL*** representation also includes the use of the bar model.

***ABSTRACT RECORDING***: this is the ultimate mode but should not be rushed at the expense of true ***CONCEPTUAL UNDERSTANDING***. Scaffolds, such as missing box calculations, can support children’s transition towards ***ABSTRACT***.